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Article

Not Finitude but Countability: Implications of Imagination Positing Countability in Time

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Abstract: In this article, we will show how imagination and time are two sides of the same coin. To explain this, we require that imagination posits countability of alternatives. A countable set of alternatives can be sequenced on a timeline, for example the thinking human's past, or it can be expressed as a countably infinite set of cycles such as a Fourier transform gives us. At the heart of our discussion is a technical argument arising from Cantor's diagonal method. A conclusion that we arrive at is that the finite/infinite opposition, in particular in philosophy, is confusing at best. Instead we propose a countable/uncountable opposition as being a far clearer basis for understanding human imagination and as a basis for the philosophy of time. We discuss Kant, Heidegger and Gödel in this light. We draw out implications for "machine imagination," and we propose a new basis for understanding human creativity and imagination.

Key words: Imagination, time, collective intelligence, machine learning

1 Introduction

It is not far-fetched to consider a work of literary creativity—say, Joyce's *Ulysses*—as a computed object. In neural networks and connectionism, biometric principles are used to suggest new algorithms for pattern recognition and decision support. Going further than just algorithms, there have been proposals at various times for new forms of computation. Wolfram, for instance, discusses the computing of complex objects, including the universe itself.[1] Especially in observational sciences (ecology, astronomy) the term "citizen science" or even "carbon-based computing" ("carbon-based" implying large numbers of networked people) has been a growing trend in recent times. In this article, we aim to show that the (intellectual, creative, artistic) imagination in operation can be considered as a form of computation, although this is a form of computation that goes beyond current forms of computation as we will discuss.

[1] S. Wolfman, *A New Kind of Science* (Champaign, IL: Wolfram Media, 2002).

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This work employs Cantor's distinction between countable and beyond countable number systems in quite a straightforward way in order to characterize the creative, imaginative object and the process of creation of that object. As will be seen, we take the created object as a computed one. Practical implications of our work include the following:

- It is held that we will soon be able to record everything a person experiences in a lifetime (see section 1.3), and everything a person does. Does this open up the possibility for determinism in mapping human thought from human action, and if not why not?
- Diagnostic and decision support systems of various sorts are largely black boxes, and evaluation is also largely validation of outputs. In section 10 we note how restrictive and limited this really is.
- In section 11 we take issue with recently proposed views that thinking can be automated through collective, aggregated action, or by search engine-based massive correlation of information.
- In conclusion, section 12, we note how metaphor alone does not go far enough for addressing the areas at issue in this article. Hence this is an additional justification for our work as reported on here.

In this work, we establish an integral link between (i) creative thinking, (ii) countability, and (iii) time. Especially for the link between (i) and (ii), we use in a very straightforward way Cantor's Diagonal Method. Cantor's method maps one set onto another. It establishes an isomorphism between two sets, meaning that there is an identity vis-à-vis labeling between the two sets. As a byproduct the Cantor method can be used to establish whether a set is countable or not. It should be noted that the term *countability* is used in its strict sense of either finite or denumerable.

Human thinking may be taken as embracing a range of modes, e.g. problem-solving (with various classes of problem); unconscious response; response through rote learning and repetition; and so on. What we are concerned with is thought, a pattern of human behavior, characterized by diachronic insight and synchronic novelty. A major subtheme for us is whether, or to what extent, thought as a pattern of human behavior also holds for complex systems generally.

Creativity is considered here only in its most basic form as a thought, or its most undifferentiated form as an artistic or other reflective product, taken as a unit.

1.1 Creative Thinking: Creative with Respect to What Exactly?

By thinking we intend here not spontaneous and reactive; but instead insightful, imaginative and novel, and recognized as such. Here we address how

human thinking is considered by the thinking subject as insightful, imaginative or novel. Our discussion is laid out as follows:

Ongoing ideas generated by the thinking subject are considered by him/her as new. Flashback to times past is an exceptional state. Even there, an exact reinsertion into some previous state of thinking is clearly impossible. Novelty of thinking is what we focus on here. Based on thinking, how do we advance irreversibly and hence what we term "forward" in time? We rephrase this question: how is each newly generated unit of thinking potentially *new*? That is, how is it different for us, as thinker, from previous units of thinking? As thinker we are aware, i.e. conscious, of the newness of our thoughts.

We show that such forward movement in time can be related to Cantor's diagonal method if the following conditions are met. The thinking action comprises some unit, and can be compared to other units. We do not care what these units are so long as we can consider them units of thought. A unit of thinking can be a problem-solving approach (covering also such terms as used in mathematics as *Ansatz* or *démarche*), or an act of creative thinking. Included in the latter is an act of sustained thinking and reflection that results in a work of art or literature or science. All of these are relevant for us in that they define straightforwardly units of thinking. We can replace any unit of thinking with an idea so long as we take the latter as having non-negligible extent in time. We have less control over dreams, in the sense of auditing them, and so we set aside, in this article, any consideration of their being also taken as units of thinking.

We use units of ideas as our observed objects of study. Let us furthermore consider just one human individual, separate from all others. We do this in order to limit the focus of our work, and will return later in section 4 to the social or collective aspect.

By considering units of thinking, the nub of our argument is that (i) novelty is relative to past units of thinking; that (ii) past units are definable in a countable basis (mathematical sense), i.e. in some countable way; and that (iii) a unit of thinking is novel precisely because any such unit can be shown to be *not* defined from a countable basis set.

To invoke Cantor's diagonal proof in this argument we associate past units of thinking with the commonly understood timeline, by requiring (i) a countable basis to be used on the timeline, and (ii) the set of past units of thinking to map onto the range of a finite function (we can call this mapping "thinking") from the domain given by the countable basis. Therefore, we use a bijection between ideas and countable basis of the timeline.[2]

The timeline is defined by one's own or any arbitrary external contextual history.

We then look at other bijections with the countable basis of the timeline. This furnishes us with a theory of creative thinking, with quite different units of thinking (work of music, of writing, scientific result, etc.).

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[2] See M. Davies, *The Universal Computer: The Road from Leibniz to Turing* (New York, NY: Norton, 2000).

In summary, here is another view of our discussion. Firstly we take novelty of situation. Novelty of situation is not a singularity but has extent. We will say it has extent in time. Novelty implies countability of all possible alternatives to our novel situation. The latter—all possible alternatives to our novel situation—is structured; it is sequential for the most part although other geometries or topologies could well be considered.

1.2 Demarcating, Recording and Storing Thinking

Consider how we determine units of creative thinking. Chafe, in analyzing verbalized memory,[3] considered the units of: (i) *Memory* expressed by a *story* (memory takes the form of an "island"; it is "highly selective"; it is a "disjointed chunk"; but it is not a book, nor a chapter, nor a continuous record, nor a stream); (ii) *Episode*, expressed by a *paragraph*; (iii) *Thought*, expressed by a *sentence*; and (iv) A *focus*, expressed by a *phrase* (often these phrases are linguistic "clauses"). Foci are "in a sense, the basic units of memory in that they represent the amount of information to which a person can devote his central attention at any one time." The "flow of thought and the flow of language" are treated at once, the latter proxying the former, and analyzed in their linear and/or hierarchical structure by Chafe and others.[4]

We have here various ways of defining or determining units of thinking. If all were finite, then combinatorial arguments would be appropriate and sufficient. However many aspects of capturing the notion of artistic creation, not to mention creative thinking, may be taken as infinite. An essential message in this article is that considering an infinite set of creative thoughts or outputs in an individual life is quite mundane. By definition of creative thinking we will assimilate units of creative thinking as objects of analysis to some infinite set. This makes our task more enigmatic because it is no longer a straightforward finite, combinatorial case. The intent of this article is to show how easily we can accommodate creative outputs defined in a (countable or denumerable) infinite way. We require that our past, given or established, set of creative outputs must be countable in the mathematical sense of finite (which we ignore as uninteresting) or denumerable.

Would our argument be weakened by noting that finiteness is in question, rather than countability, since after all the number of seconds (or other unit of time) in James Joyce's life, say, was a moderate sized integer (hence natural and finite) number? We avoid this issue by just considering a

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[3] See W.L. Chafe, "The flow of thought and the flow of language," in *Syntax and Semantics: Discourse and Syntax*, ed. by Talmy Givón, vol. 12, 159–181 (Amsterdam: Academic Press, 1979).

[4] For more general text, we can consider segmentation. Examples of text segmentation to open up the analysis of style and structure include Y. Bestgen, "Segmentation markers as trace and signal of discourse structure," in *Journal of Pragmatics*, 29 (1998), 753–763; B.J. Grosz and C.L. Sidner, "Attention, intentions, and the structure of discourse," in *Computational Linguistics*, 12 (1986), 175–204; and B.J. Grosz, "Discourse structure, intentions, and intonation," in *The Languages of the Brain*, ed. A Galaburda, S Kosslyn and Y Christen (Cambridge, MA: Harvard University Press, 2002).

transform like the Fourier transform of any signals of Joyce's timeline: say, signals representing Joyce's brain activity. This introduces a new basis, in which Joyce's creative activities can be mapped. A Fourier basis, built from sines and cosines, could perhaps be interpreted as the cyclical phenomena and events underlying the Joycean timeline. The important property of a Fourier transform that we should note however is that it is defined in a *countable*, and not necessarily finite, basis. So this strategem from signal processing shows very clearly that countability is what is important, and not just finiteness.

1.3 Towards a Critique of the Storage of All of One's Thinking and Memory

One can adopt a mechanistic perspective on creative thinking by reducing it to a purely combinatorial arrangement of basic building blocks. James Joyce's *Ulysses* has 304,414 different words and (we have found) 28,631 unique words. We can go on to consider the totality of Joyce's work as so many units such as words. In a similar vein, the work of Shakespeare, according to Buckley, amounts to under one million words, and can be spoken in 70 hours.[5]

Dix considers how a person can be monitored by a video camera for their entire life. The amount of video data, at some standard resolution rate, for 70 years or 2.2 x 109 seconds, is to an approximation 27.5 terabytes. Let us pose the question of the complexity of a human life, expressed as this particular 27.5 terabytes of information. The "Memories for Life" is a proposed grand challenge of preserving intact and easily accessible all memories of one's life.[6] A similar aim motivates the MyLifeBits project, entailing the constant recording of image, video, biomedical signal, thought and such data.[7]

Basing himself approvingly on a publication by R. Kolisch in 1943, musical and cultural theorist Adorno considered:

> . . . the basic characters to which the types of Beethoven's tempi correspond. In this way, [we arrive] at a discrete number of such basic characters and tempi. At first, the result is shocking; it seems a bit mechanistic and overly mathematical in relation to Beethoven's gigantic oeuvre. But if you turn the tables, . . .you will find that great . . .music actually bears some resemblance to a puzzle. The

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[5] See W.F. Buckley Jr., "Variations, review of *Johann Sebastian Bach: Life and Work,*" trans. by J. Hargraves, Harcourt, in *New York Times, Sunday Book Review* (3 Dec. 2006).

[6] See A. Dix, "The ultimate interface and the sums of life?," in *Interfaces*, 50 (2002), 16 and K. O'Hara et al., "Memories for life: a review of the science and technology," in *Journal of the Royal Society Interface*, 3 (2006), 351–366.

[7] See G. Bell, "MyLifeBits Project, Microsoft BARC Media Presence Group," http://research.microsoft.com/barc/MediaPresence/MyLifeBits.aspx (2008).

> movements of the greatest composers are based on a discrete number of *topoi*, of more or less rigid elements, out of which they are constructed. . . .Music represents itself as if one thing were developing out of the other, but without any such development literally occurring. The mechanical aspect is covered up by the art of composition . . ..[8]

Adorno's discussion continues with a reference to a similar picture in relation to how "Similarly, with a certain amount of naïvité, the great philosophical systems beginning with Plato have had recourse again and again to such mechanical means . . .."[9]

In this article one of our objectives is to demonstrate that a mechanistic and finite view of *creative* thinking is untenable.

For James Joyce, what constituted the originality of *Ulysses*? A finitary and combinatorial view leads to a proverbial blind monkey with a typewriter eventually, after a very long time, producing the work. Our view is very different. Take Joyce's work *Ulysses* as a set of motifs. Order all possible precedents in regard to the motifs. Our view is that an ordered set of all such precedents in a countable base is feasible and indeed inherent in the very concept of *novelty* of a work like *Ulysses*. The thinking, and still imaginative and working Joyce, could step outside the countable set of past achievements.

Let us look now at this in more detail.

2 Creative Thinking in Time

2.1 Introduction

A product—e.g., a book or some other work of art, etc.—is an object that has finite characteristics. It is the finite and limited aspects that are of interest here. We generalize any such object to a thought. Take for example the thought that is the basis for a concept for a television series. It too is considered in terms of its limited, and hence finite, aspect.

Time is usually considered in science as one dimension, the real number line, constituting one axis of the n-dimensional reals. We consider a countable covering of the real timeline. One example is just to look at the natural numbers, 1, 2, . . ., on the real timeline. A trivial way of doing this is to sample the reals by using the naturals. Each natural can be mapped onto the reals, but the reverse is not true. We have an injective but not an onto mapping. By representing the timeline as the naturals, it results that time is countable (viz., finite or denumerable).

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[8] T. Adorno, "Difficulties," in *Essays on Music*, Selected, with Introduction, Commentary, and Notes by R. Leppert, new translations by S.H. Gillespie (Berkeley, CA: University of California Press, 2002), 667.

[9] *Ibid.*

Imagination is taken here as resulting in novel thoughts. A *novel* thought is a new one, or one that has not been met with before. We call this creative thinking. Since we want to associate creative thinking with some arbitrary countable version of the timeline we view creative thinking in terms of some arbitrary units of creative thinking.

Heidegger indicates: "knowledge . . .[is] the power of imagination and 'time'," [10] but without considerable further elaboration this is obscure. In the sequel, we will throw light on this.

2.2 Creative Acts Referenced to an Ordered Set of Natural Numbers

In this section we want to establish past creative accomplishment—in literature, say, but this can also apply to music or science and so on—as countable or denumerable. This implies that we can list the creative accomplishments in whatever way we choose and list as first, second, third, and so on. That is to say, we find a bijection with the natural numbers.

Consider a scenario where a subject (human individual) writes a text, *t*, and we can associate a unique natural number with this, *N(t)*. This number can be arbitrarily large in order to be unique. It is a hash key, in database or information retrieval parlance. Elsewhere, I consider mapping texts—literary, literary, technical, etc.—first into multidimensional spaces and then mapping these spaces into others that are suitable for visualizing and interpreting the data. Such mapping techniques are also very relevant in this context. It is useful for us to note here that our framework could alternatively be set up with *N(t)* being a point in a high dimensional semantic space.[11]

We proceed now to a life's work which in our simplified scenario comprises a sequence of texts, *t(1), t(2), . . .t(z)* where *t(1)* is the first text that this author has written, and *t(z)* is the last one. Each such work can be mapped uniquely, e.g. via completion date, onto time points in the subject's life, so we can easily write instead this sequence of time points as: *t'(1), t'(2), . . ., t'(z)*. With each text is associated our encoding of the work, which we can index either by object or in time: that is, the sequence *N(t),* or the sequence *N(t')*.

For any subject the domain set of created objects, *t*, or of times in a subject's life, *t'*, is finite and as such uninteresting. This scenario fits well into a mechanistic view of creative work, where one work is finalized and later the next work begins. Then it becomes finalized in turn, and later yet another work is undertaken.

Let us now generalize this. We take some other countable basis. Take a Fourier transform basis. Then any function can be written as a countable sum of sinusoidal components. To provide a rationale for a Fourier basis, consider

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[10] M. Heidegger, *Kant and the Problem of Metaphysics*, Fifth edition, enlarged, translated by R. Taft (Bloomington, IN: Indiana University Press, 1997), 209.

[11] See F. Murtagh, *Correspondence Analysis and Data Coding with R and Java* (Boca Raton, FL: Chapman and Hall/CRC Press, 2005).

this: creative creation is related for a subject to some cyclical backdrop, such as an early morning creative frame of mind; or an annual seasonal event such as discussions and debates with research colleagues at an annual summer school; or one could link creative acts to economic cycles or socio-technological waves. We have no dearth of possibilities for giving a meaning to a countable basis like the Fourier transform on the real timeline. Let us proceed now with such a new countable basis, the domain of which we will denote by *s(1), s(2), . . .* where now *1, 2, . . .*is the denumerable and not necessarily finite sequence set.

Each creative work in a subject's life, in the scenario denoted *N(t),* can now be represented in this new basis. Let us represent the creative work in this new basis as *s(N(t))*. If we aggregate all creative works in our subject's life, say just by adding them together, we get an aggregated collection of *s(N(t))* for all the creative works, *t*, that we consider.

The set of all of a subject's creative works, which we will denote as *T*, is therefore structured on a *countable* or *denumerable* set.

From Levy we have the following definitions. A set is *denumerable* if it is equinumerous with ω. ω denotes the least infinite ordinal. It is the set of all finite ordinals. This is how we formally state that we are dealing with infinity, in the particular sense of the natural numbers. A set is *countable* if it is either denumerable or finite.[12]

2.3 Accomplished Versus Novel Creative Items

As noted in the previous section, the set *T* of accomplished creative items is taken in a countable basis. Let *T'* be the set of *novel* creative items. Each novel creative item is a unit of creative thinking. Suppose *G* is a bijection of *T* on *T'*. Construct a novel creation, a member of *T'*, and let this novel creation be indexed by *r,* i.e. *t(r)*. By Cantor's diagonal method[13] we ensure by design that the new member of *T'* is different from each and every member of *T*. The basis set therefore from which each member of *T'* is defined, and that is indexed by *r*, is not countable in the sense that it goes beyond the countable set of already created items.

Therefore there is *no* necessary bijection *G* of *T* on *T'*. Or, alternatively expressed, *T'* will not be in the range of *T'* under *G*. Our assumption of a bijection *G* of *T* on *T'* is not valid.

T is both countable and ordered. In the diagonal method, we must construct *t(r)* in our *s* basis, with first element different from the first element of *t(1)* in the *s* basis; with second element different from the second element of *t(2)* in the *s* basis; and so on. So the set *T'* of new creations is not countable, there is no bijection *G*, and we have exhibited *t(r)* to be a genuinely new unit of thinking, that differs from any existing countable set.

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[12] Cf. A. Levy, *Basic Set Theory* (Mineoa, NY: Dover, 2002).

[13] A short technical note is available from the author that elaborates on the presentation in section 2.3.

We ignore the finite context here because it is overly simple: if all in our system were finite, we would just be saying that imagination implies a thought that we number *N+1* whenever there are *N* other thoughts pre-existing it or co-existing with it. As such, thought *N+1* is different from the *N* other thoughts, and satisfies our criterion of novelty. Everything is finite and remains so. Everything is trivially countable. Every thought can be associated a unique label, or a sequence number. We could store all thoughts of a given individual, assuming sufficient storage, and assuming the finiteness of a thought.

2.4 Imagination Implies (i) Transcendence, and (ii) Countability in Time

From a mathematical point of view, we are saying the following. Imagination implies an outcome (product, thought) that is beyond a countable set. The countable set of historical or other alternatives is a general view of such alternatives, and can be formulated (as we have seen) through expressing an act of imagination in terms of some countable basis. Imagination thus *transcends* the countable, timeline-associated alternatives.

Our way of linking creative act to countability is via the intuiting of time. On the historic, and accomplished, timeline, we have transformed, or so to speak interpreted, past acts such that they have as their basis a countable set. In this somewhat indirect way we therefore link creative activity to time. We make use of a countable or denumerable basis on the subject's (or as will be seen in section 4 below, group or the societal) timeline.

This countable or denumerable view of one's or a group's timeline is what we next examine. We consider countabilty not of, but rather in, time.

3 Kant's Theory of Time, Finiteness and Countability

3.1 Kant's Theory of Time Through the Prism of Heidegger

There is much in Heidegger that is very germane to our discussion. He targets for example: "a transcendental, ontological determination of time . . .within which something like the permanence of the substance is constituted for the first time." Time structures our experience, just as experience posits time: "time is not just a setting in which experiences play themselves out."[14]

One significant aspect of where we differ from Heidegger is that the finiteness is not what we find to be important. Instead we believe from our argument in earlier sections, countability is what is necessary to consider. For Heidegger as for Kant, "finitude is placed at the point of departure for transcendence": we disagree—countability is the point of departure. The "finitude of knowing" is not true to life in our view. "Thrownness (*Geworfenheit*),

[14] Heidegger, *Kant and the Problem of Metaphysics*, 198.

the ground for the finitude of knowing" is also lacking in clarity. No: for us, imagination and creative thinking transcends the *countability* of selfhood and not just (and rarely ever in fact) its finiteness. The reality of experience is far too complicated to be considered as finite. As we have discussed in previous sections we can easily formulate a view where a countable basis expresses very well a sophisticated and realistic view of experience.[15]

The marshalling of things by time, and the fact that these are not finite but rather countable, is what establishes the *novelty* of thinking. In the same moment, it establishes the thinker too. Insofar as the thinker is consciously thinking, the thought is novel. That is, the thinker is distinguishing the thought from a countable set of baseline thoughts from the past. "Sense means finite intuition," says Heidegger. No, we disagree: sense means countable intuition.

We do agree strongly with Heidegger that time is a major player (we do not consider e.g. space here) in what "makes the mind into a mind." "Time and the 'I think' no longer stand incompatibly and incomparably at odds, they are the same." We have found that novelty of thinking and countability are two sides of the same coin. Let us look at novelty of thinking in terms of transcendence. "If the transcendental power of imagination, as the pure forming faculty, in itself forms time—i.e., allows time to spring forth—then we cannot avoid the thesis . . .: the transcendental power of imagination is original time."[16] So, imagination is transcendental; imagination creates time; time, in an abstract (here: mediated) sense, defines imagination. Heidegger again: "time takes part essentially in the innermost essential structure of transcendence." Or again: "the transcendental power of imagination allows time as sequence of nows to spring forth, and as this letting-spring-forth it is therefore original time." And: "the forming [*Bilden*] of the imagination [*Einbildung*] is in itself relative to time. Pure imagining . . . must first of all form time." "In the periodicity of the seasons, in the rhythm of the phases of life and age, the power of time is made evident." Heidegger goes on to consider myth and religion in terms of how they structure time. Similarly the thinking mind induces "cosmic time," "calendrical regulation" and "ethical obligation," all imposing a time order on things, and indeed (in the prism of myth) "an order of destiny." "Numbering and the relations of number" follow, or sit alongside these other views of the structuring aspect of time.

Novel thinking involves no latency or time lag: "the transcendental power of imagination . . .is essentially spontaneous receptivity and receptive spontaneity." The lack of any latency or of any possibility of latency is a characteristic of imagination in our sense. This counters any ultimate machine intelligence as spurious because this looks exclusively at outputs and makes no distinction between human-like behavior other than through outputs. The

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[15] As an aside, Badiou is in keeping with our view: "This is what we will call the *transcendental*: the entire apparatus which must be presupposed in order to be able to think difference within appearance." Moreover, "the finitude of its act and the infinity of its being." *Theoretical Writings*, edited and trans. by Ray Brassier and Alberto Toscano (London, UK: Continuum, 2004), 117, 187.

[16] Heidegger, *Kant and the Problem of Metaphysics*, 131.

Turing test exemplifies this well (i.e., a machine or any system is intelligent if there is no difference between its output and the output of any intelligent being). Computer reasoning has nothing whatsoever to do with imagination in our sense (viz., defined by countability). Instead it has to do with a very different class of problem solving or decision making.

3.2 Gödel's Path From Finiteness to Countability

We briefly discuss Gödel's path from (i) the finite, capable of being exhibited; (ii) implying the notion—in his terms—of intuitive, "passing stepwise" from one case to the next, i.e. inductive; and (iii) on to the computable.

3.2.1 Gödel on Kant's View of Time

Gödel took a somewhat distanced baseline view of Kant: "a general feature of Kant's assertions that literally understood they are false, but in a broader sense contain deeper truths."[17] More to the point are that the issues dealt with here, viz., Gödel's views, are somewhat unclear. Faced with the views of some authors that Gödel's views were "unstable," "unsettled," "vacillating" and even incoherent, Feferman, nonetheless, argues for a level of coherence ultimately to be found in Gödel's views.[18]

Gödel recognized that for Kant, time is "not 'something existing in itself' (i.e., a separate entity besides the objects in it) nor 'a characteristic or ordering inherent in the objects' but only a characteristic inherent in the relation of the objects to something else."[19] With reference to relativity theory, the idea that time "as its most essential characteristic...consists [in the traditional view] of a one-dimensional system of points, isomorphic with a straight line, in which every happening in the world has a definite place." Instead, for Kant, time as "a one-dimensional temporal ordering of the events" is the case, and is "relative to the perceiving subject or more precisely its 'sensibility'."[20] And again: for Kant, "that temporal properties represent certain *relations* of the things to the perceiving subject appears from many passages in his writings." Gödel proceeds to give a range of examples from Kant.[21]

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[17] K. Gödel, "The modern development of the foundations of mathematics," in *Kurt Gödel, Collected Works, Volume III, Unpublished Essays and Lectures*, ed. By S. Feferman et al. (1995a (Oxford: Oxford University Press, 1995), 385. Also see P. Yourgrau, *A World Without Time. The Forgotten Legacy of Gödel and Einstein* (New York, NY: Allen Lane, 2005), 106.

[18] See S. Feferman, "Lieber Herr Bernays! Lieber Herr Gödel! Gödel on finitism, constructivity and Hilbert's program," *Dialectica*, 62 (2008) 179–203.

[19] K. Gödel, "Some observations about the relationship between theory of relativity and Kantian philosophy," in *Kurt Gödel, Collected Works, Volume III, Unpublished Essays and Lectures*, 247.

[20] *Ibid*., 248–249.

[21] *Ibid*., 249.

3.2.2 Finiteness, Countability and Computability

Gödel addresses how finiteness, associated for example with inductive proof, is intuitive: "finite mathematics is defined as the mathematics in which evidence rests on what is *intuitive*." And (emphasis in original):

> we cannot acquire . . .knowledge *intuitively* by passing stepwise from smaller to larger ordinals; we can only gain knowledge abstractly by means of notions of higher type.[22]

Turning to the infinite or non-intuitive, there is a leap from concrete things to virtual objects:

> . . . by abstract (or nonintuitive) notions we must understand those that are essentially of second or higher order, that is, notions that do not involve properties or relations of *concrete objects* (for example, of combinations of signs), but that relate to *mental constructs* (for example, proofs, meaningful statements, and so on); and in the proofs we make use of insights, into these mental constructs, that spring not from the combinatorial (spatiotemporal) properties of the sign combinations representing the proofs, but only from their *meaning* .[23]

Finite, for Gödel, is either constructive—further defined as capable of being exhibited, or constructed; or finite "which requires in addition that the objects and facts considered should be given in concrete mathematical intuition." Gödel continues:

> This, as far as the objects are concerned, means that they must be finite space-time configurations of elements whose nature is irrelevant except for equality or difference. (In contrast to this, the objects in intuitionistic logic are meaningful propositions and proofs.)[24]

Gödel refers to finitary mathematics—the "finitary attitude" as he characterizes this—as either constructive or intuitive. The latter is ."... the specifically *finitistic* element [that] requires in addition that the objects and facts considered should be given in concrete mathematical intuition."[25]

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[22] K. Gödel, "On a hitherto unutilized extension of the finitary standpoint," (originally 1958 and 1972), in *Kurt Gödel, Collected Works, Volume II, Publications 1938–1974* (Oxford: Oxford University Press, 1999), 241, 243

[23] *Ibid.*, 241.

[24] Gödel, *Kurt Gödel, Collected Works, Volume III, Unpublished Essays and Lectures*, 274

[25] Gödel, *Kurt Gödel, Collected Works, Volume II, Publications 1938–1974*, 245.

> It is the second requirement [viz., the intuitive viewpoint] that must be dropped. . . .instead, one can use . . .a certain concept of a *computable function of finite type over the natural numbers* and some very elementary axioms and principles of construction for such functions.[26]

We summarize: the function is of finite type. We have a computable function over the naturals.

So Gödel noted how we can go beyond the intuitive property. He retained the constructive property though. He even retained an infinitary property. Functions on a denumerable set were mobilized. Finite, per se, is of some use only for us. We require countability and not just finiteness.

For Gödel there is a "plausibility that all things conceivable by us are denumerable." In the same article, he indicated further:

> 'definability in terms of the ordinals', even if it is not an adequate formulation for 'comprehensibility by our mind', is at least an adequate formulation in an absolute sense for a closely related property of sets, namely, the property of 'being formed according to a law' as opposed to 'being formed by a random choice of the elements.[27]

4 Application to Societal Thinking: The Case of Archaeological Evolution

We now apply the results established in this article to archaeological time and to human evolution in that framework, following the well-argued presentation of Gamble.[28]

Gamble critiques the successive revolutions (neolithic, sedentary, etc.) view of human evolution. His fundamental idea is that *material metaphors* are of enormous importance. "My central point is that devices such as metaphor, metonymy, synecdoche and analogy can be expressed materially as well as linguistically." Metaphor is "experiencing one kind of thing in terms of another." "Metaphor establishes a common understanding between things that are as alien as chalk and cheese."[29] For Gamble metaphor is material and not just linguistic. So speech, often based on metaphor, is seen by him as a special type of artifact: ."".. a mimetic style of thought . . .models the body, and the objects that are like the body is our key skill rather than language, important as that was for elaboration and undoubtedly social extension."[30] Then,

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[26] *Ibid.*

[27] Gödel, *Kurt Gödel, Collected Works, Volume II, Publications 1938–1974*, 152

[28] See *Origins and Revolutions: Human Identity in Earliest Prehistory* (Cambridge, UK: Cambridge University Press, 2007).

[29] *Ibid.*, 88.

[30] *Ibid.*, 273.

> . . . artifacts, the archaeologist's bread and butter evidence, act as material metaphors for . . . inner identity. . . . Artifacts are much older than words. Tools and techniques have always had a metaphorical relationship with the . . .body and identities have been formed from this interaction.[31]

Gamble uses this stance to counter the view of demarcated phases of human evolution, each with a well-defined revolution (for instance, to agriculture); ."... experience articulated through novel material metaphors" is how he proposes that human evolution should be viewed. So here we have our schema quite clearly: a material metaphor is new so long as it can be counterposed to a countable set of past material metaphors. The latter is situated in evolutionary or archaeological time, if only because we can envisage past material metaphors referenced to a countable set.[32]

On archaeological time we define any arbitrary countable basis. It is feasible for us too to chop up historical time into segments, even if there are untold numbers of ways to do this. What we require is that past material metaphors be countable and/or defined through a finite function defined on a countable basis. Then by construction we can show, as in section 2.3, that the new material metaphors do not belong to the countable set (by definition of being new). In that sense the new material metaphors can be said to be uncountable. This establishes their innovative impact. It also simultaneously establishes societal time or history.

Gamble continues: "It is the importance of material metaphors, as simple as a stone tool, rather than just the forms of social relationship, that have to be appreciated as the basis of a relational identity." So the mediation of tools and techniques becomes crucially important in the relationship between people. Furthermore society is genuinely governed by its artifacts and the organisation of society is secondary. Society's consciousness comes from its metaphors, derived from its artifacts, and the material metaphors give rise ultimately to forms of organisation. It is not forms of organisation that are imposed from the start. "Cognition is essentially a process of seeing something *as* something and this is the core of metaphorical understandings":[33]

> . . . metaphors, far from simply facilitating understanding as they are usually presented, must now be understood as governing the way we think. . . .our entire use of concepts depends on metaphor . . ..[34]

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[31] *Ibid.*

[32] Cf. Gamble, *Origins and Revolutions: Human Identity in Earliest Prehistory*.

[33] *Ibid.*, 68. Also, C. Tilley, *Metaphor and Material Culture* (Oxford: Blackwell, 1999).

[34] Gamble, *Origins and Revolutions: Human Identity in Earliest Prehistory*, 66.

The "authority of commonplace material metaphors . . .[organise] the world of experience" and "the epitome of civilised society owes more to the body and its material proxies and less to the triumph of the mind."[35] As an example, the linguistic metaphor "network" was originally used for thread in the French silk trade, and the term "elbow" has given rise to such metaphors as "give the elbow," "elbow room," "elbow one's way." Gamble's main *propos* in his book lies with two root metaphors: instruments and containers, that proxy respectively the body's limbs and the trunk.

> Instruments and containers have great antiquity and have always been referenced to the body. . . .material proxies for the containers and instruments of the body pre-date their linguistic utterance.[36]

It is a "tradition" to place changes in material metaphor—leading to animal domestication for instance, or development of the blade—on a timeline. As we see in this article, there is another reason: namely, to establish their novelty. Change takes place, for example, when "the authority of containers came to eclipse that of instruments, and with that came the possibility of agriculture." The timeline, we see, is inherent to change (in the sense of innovation). It is not just an abstract (or a fortiori integral, inherent) organization principle.

Furthermore as seen in this article, we can easily admit a more complex and colored view of establishment of novelty, by having an indirectly defined countable set of alternatives that are related to the timeline by means of a countable basis on the timeline.

Experience of—and via—the body gives rise to artifacts that establish our concepts; and metaphors link those concepts together. As Gamble states:

> What we understand by meaning cannot just be a mental activity but needs to incorporate the body-whole as well as the world it inhabits. Now, to understand this world we need those metaphors that establish the links between concepts based on the experience of the body.[37]

We know that new material metaphors are new because we can enumerate potential alternatives. The personal, experience-based, most meaningful way to do this enumeration is not to put metaphors side by side with grains of sand or galaxies in the cosmos. Rather, it is to put them on a timeline, and then establish a countable basis. The latter is implicit in any historical or prehistorical periodization, in whatever optic—personal, group or social.

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[35] *Ibid.*, 6, 280.
[36] *Ibid.*, 110.
[37] *Ibid.*, 103.

5 Mathematics and Thinking Betoken More Than Just Metaphor

Our work is quite separate from seeking physiological bases for counting or creative thinking.[38] Seeking brain structures for creative thinking, counting, and understanding of time, all come later, in our view. We need to be clear about the inherent linkage between these seemingly quite distant aspects of human activity. That has been our aim in this article. Similarly Lakoff and Núñez seek to ground arithmetic and then mathematical ability in metaphor.[39] Our aim is to go much further: to show, in fact, that creative thinking, counting and time are all inherently related. Two issues, in particular, follow from these remarks.

Firstly, Dehaene's cogent and clear expression of the cognitive science viewpoint is not acceptable to us, since it is (in our view) hugely mechanistic. In Duhaene, the "search for universal laws in psychology" is tabled:

> The confrontation of ancient philosophical questions, often asked with acuity by Plato, Kant, or Descartes, by the new technologies of the behavioral sciences of neuroimaging and of mathematical modeling creates a friction particularly favorable to the emergence of new knowledge.[40]

While the "engineering" and instrumental use of cognitive psychology is not in doubt, nonetheless we should be wary of over-generalizing from such an optic. The "new technologies" only go so far in addressing our need to know the limits of how much we can know.

A second elaboration of our viewpoint concerns the use of metaphor in Lakoff and Núñez. We supported strongly the Gamble metaphor (and artifact) optic. A clear undercurrent however in this article has been running counter to Lakoff and Núñez's use of metaphor as a basis for mathematics. Instead, as for Kant and (in our view) Gödel, we see a great deal of mathematics as being a priori. In this article an aim has been to show that countability, time and creative thinking are inherently related.

Badiou, who has championed mathematical philosophy, has time, infinity and truth as cornerstones. Badiou says:

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[38] For instance, S. Dehaene, "A few steps towards a science of mental life," in *Mind, Brain, and Education*, 1 (2007) 28–47 and V. Izard et al., "Distinct cerebral pathways for object identity and number in human infants," *PLoS Biology*, 6(2):e11 (2008).

[39] G. Lakoff and R. Núñez, *Where Mathematics Comes From: How the Embodied Mind Brings Mathematics into Being* (Basic Books, 2001).

[40] Dehaene, "A few steps towards a science of mental life."

> . . . the truth process is not formally temporal. While we have successive inquiries, I do not at all place that in a temporal series, but it is a possibility. I have always said that there is an event and after an event, a truth process. It is a construction of a new time, but I have not done much work to make this concept more precise. But the general idea is really that there is finally something temporal in the very process of truth, not in time itself but the generation of a new time.[41]

In this article we are not concerned with truth, nor a fortiori with logic, but rather with what we term creative thinking.

Creative thinking, as we have argued, is part and parcel of counter-position to alternatives. Such alternatives by being placed on, or otherwise related to, a timeline become countable for us. Countability involves either a finite set, which is not of interest here insofar as it is quite trivial; or an infinite set which is denumerable, i.e. offering a bijection to the natural numbers. Through having a countable set, we have exhibited a new element that is not in the countable set. Consequently novelty goes hand in hand with uncountability. The infinite set of novel elements is greater than the infinite set of past exemplars.

Let us put the spotlight on our exhibiting of a new element. Our sole objective in this work was to draw out the implications of doing this. In practice any such new element will take account of past exemplars on the timeline. Furthermore any such new element can come about through "negotiation" with past exemplars or through rupture. Gamble emphasizes new material metaphors arising through negotiation, and then how such negotiation allows identity of humans and/or of societies to emerge. This is an interesting and important point: i.e. identity or self-awareness emerging from novelty. It is an elaboration of our more basic argument in this article, viz. that the human act of creative thinking creates time. The sense in which we understand this creation of time is that there is arranging or choreographing of acts, thinking, or such alternatives.

Let us turn to the situation of non-novelty, i.e. a more "automated pilot" behavior. Merleau-Ponty, cited by Gamble,[42] characterizes habit thus: "We say that the body has understood, and habit acquired when it has absorbed a new meaning, and assimilated a fresh core of significance." Gamble adds: "The routines which contribute to *habitus* are part of our practical, rather than discursive, consciousness." Habitus embraces the "recurrent patterns of artifacts in time and space that give a distinctive form to the past." Such "recurrent patterns" are not the same as our countability requirement (which

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[41] A. Badiou, "New horizons in mathematics as a philosophical condition: an interview with Alain Badiou," in *Parrhesia, A Journal of Critical Philosophy*, Issue 3 (2007) 1–11.

[42] Gamble, *Origins and Revolutions: Human Identity in Earliest Prehistory*, 93.

we use to establish novelty). They could be, for example, the layers of experience and artifacts that are the raw materials for archaeology.

The importance of the "stable dispositional tendencies" (Merleau-Ponty), termed habitus, cannot be undervalued. As quoted from Gamble at the end of section 4 above, "What we understand by meaning cannot be just a mental activity but needs to incorporate the body-whole as well as the world it inhabits." As an example: "Brains do not tell the feet what to do. Indeed our toes are as en-minded as our brains are em-bodied."[43]

We see how on the one hand we have discursive (*pace* Gamble, possibly even violent) change leading to identity formation. Or we have a different context, effecting thinking even if we do not characterize it as creative. The form of thinking is assimilated, maybe reactive and automated, but, as we have noted, also layered, patterned and structured. Unlike the transcendence involved in creative thinking or its homolog in archaeology, here there is something which is not present in the patterning and structuring of past time. What is in past time is with reference to the countable basis that we have shown to be a crucial part of our understanding of ourselves and of our world.

6 Concluding Remarks

The work in this article opens up further issues, the most pressing of which is study of the nature of the imaginative leap involved in creative thinking. Some of what is involved here is described by Badiou in the following terms:

> I cannot accept Heidegger's account of the differences between the first and second editions of the *Critique of Pure Reason*. For Heidegger, Kant retreated 'from the doctrine of the transcendental imagination'. According to Heidegger's exegesis, the 'spontaneous impetus' of the first version posited the imagination as that 'third faculty' (beside those of sensibility and understanding) providing a basis for the regime of the one and thereby guaranteeing the possibility of ontological knowledge. Heidegger reproaches Kant for failing to go further in exploring this 'unknown root' of the essence of man and for reducing the imagination to a mere operation of the understanding. Kant, he says, 'perceived the unknown and was forced to retreat. It was not just that the transcendental power of imagination frightened him, but rather that in between [the two relations] pure reason drew him increasingly under its spell'. In my opinion [continues Badiou], Kant's decision not to resort to the positivity of a third faculty

[43] *Ibid*., 103.

> (the imagination), his reduction of the problem of the one to that of a mere opposition of the understanding, testify to his critical intransigence and his refusal to concede anything to the aesthetic prestige of the ontologies of presence. The 'prestige of pure reason' may well be another name for this intransigence when faced with the great temptation.[44]

Overall, we could say that the earlier part of our article has resonated with the position of Henri Bergson's pure duration. The latter part of our article has been reminiscent of Paul Ricoeur's notion of the metaphor.[45]

There is a great deal to take further in our work. Our ultimate motivation for this work is twofold, in relation to very contemporary problems of human existence and society. In the "information universe" and the "data universe" within which we exist, (i) what is information, and (ii) what is identity? Imagination, that has been our focus in this article, is fundamental to (i) our reflective and/or reactive interaction with information, and (ii) the identity that we become.

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[44] Badiou, *Theoretical Writings*.

[45] We gratefully acknowledge an anonymous referee drawing reference in this context to Bergson and Ricoeur.

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