A Need for New Algebraic System of Logic Based on Al-Ghazali’s Reasoning

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Abstract: A qualitative nature of logic based on al-Ghazali’s teaching is reviewed. It is shown that its structure is not the same as the well-known two-valued logic which satisfies the well-known Boolean algebra. Unfortunately, it is this classical logic that has been adopted by those who discuss or use the nature of the al-Ghazali’s logic or reasoning ever since. The more appropriate logic for the al-Ghazali’s logic is the modal logic and hence the algebraic structure of this logic is reviewed. However, it is shown that even this logic, which was originally contributed substantially by Islamic scholars, in particular Ibn Sinna, and further improved by twentieth century Western scholars, is not yet fully suitable for the al-Ghazali’s logic. A new algebraic system of logic is still very much needed for characterizing the al-Ghazali’s reasoning.

Keywords: Al-Ghazali’s logical system, Islamic logic, algebra of logic, modal logic

Introduction

It is well known that al-Ghazali (who lived in the year 450 H-505 H/1057 AD-1111 AD), known in Latin as Algazel/Algazelis, provides a new system of reasoning, which requires a Muslim not to uphold determinism in an absolute sense such as that the principle of natural causality (al-sabab al-tabi’iaht) is no longer governed by certainty (due to a situation whereby, using al-Ghazali terminologies, the sunnaht or ’adaht is subjected to taqdyr or dharuraht). This has been discussed by many especially after his well-known Tahafut al-Falasifah (The Incoherence of the Philosophers) and Tahafut al-Tahafut (The Incoherence of the Incoherence) by his intellectual nemesis, Ibn Rusyd (Averroes), and still attracted many recent scholars.1 There are more

1 See the following: B. Abrahamov, “Al-Ghazālī’s Theory of Causality,” in Studia Islamica, 67 (1969), 75-98; J. al-Haqq, “Al-Ghazālī on Causality, Induction, and Miracles,” in Al-

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than 60 PhD theses (in English) on various aspects of al-Ghazali’s scholarship and at least six of them are directly on the al-Ghazali’s causality principle. The earliest is perhaps by Majid Fakhry in his 1949 PhD thesis (published in 1958). Al-Ghazali’s causality principle has also extended its application to the quantum domain as several scholars have argued strongly for the compatibility of the al-Ghazali’s causality principle with the indeterminism and uncertainty in quantum theory. In this paper, we show that this extension is untenable. However, the gist of al-Ghazali’s reasoning has already been accepted and practiced by most Muslims throughout the world by adopting the expression “insya Allah” (God willing) in every implicative statement.

As far as the claim that al-Ghazali’s reasoning is a new system of logic (formally different from the logic inherited from the Greek, the Aristotelian logic), it can be traced back to Rescher¹ (1964, 1967, 2007), and the recognition of al-Ghazali’s sophisticated criticism on induction (qiyas and istiqrā’) in a way

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very much similar to Hume, one of the celebrated British philosophers in the
18th century, is probably first discussed by Islamic Council of Europe (1982).

However, many subsequent authors such as Spade and Hintikka, Nakamura, Fauzi, Mas, Black, al-Haqq, al-Ghazali, and many anonymous and undated authors in the internet have taken it for granted. More importantly, none of them provide an explicit algebra, which could be seen as actually different from the Boolean algebra or isomorphically to the naïve set algebra which represents the classical logic (Greek logic or the Aristotelian logic) and the classical modern-scientific logic, or a non-Boolean algebra such as the von Neumann algebra or its improvements (each of which represents supposedly a quantum logic, a non-classical modern-scientific logic). It is interesting to note that even al-Ghazali himself seems to be self-contradictory when one considers his opinion on the nature and role of logic during his time as already mentioned by Shaharir and detailed by Griffel, and El Bouazzati, particularly in believing, as understood by these authors, the neutrality and universality of the Greek logic. The nature of logic as understood, elaborated, and thought to be modified or extended by al-Ghazali and studied by those writers mentioned above has not been rigorously examined based on the algebraic structure of a system of logic. In this paper we show what aspects of the Boolean logic, the quantum logic and other algebras of logic, which are still incompatible with the al-Ghazali’s logic. Thus, it shows (more explicitly than previously shown by Shaharir)
the non-uniqueness and the non-neutrality of logic and, more importantly here, al-Ghazali’s logic is indeed a new paradigm of logic and needs for a new algebra.

The Incompatibility of the Classical Logic with al-Ghazali’s Reasoning

Even though the subject matters, the events, or the statements in al-Ghazali’s universal discourse of logic can be considered as well defined to a certain extent, their occurrences are subjected to some form of uncertainty. Therefore, the classical logic (the usual logic or the traditional Greek logic) is obviously not suitable for the al-Ghazali’s logic since all the statements in the former logic involve certainty. Typical statements (involving statements A, B, and C), which are of interest in any mathematical formulation of an algebra in a system of logic, are as follows:

1. A and B, symbolically \( A \cap B \) or \( A \land B \); and a statement which involves a countably infinite number of “and”, i.e., \((A_1 \text{ and } A_2 \text{ and } \ldots \text{ and } A_n \ldots)\), symbolically \( \bigcap_{i=1}^{\infty} A_i \) or \( \bigwedge_{i=1}^{\infty} A_i \).

2. A or B, symbolically \( A \cup B \); and a statement which involves a countably infinite number of “or”, i.e., \((A_1 \text{ or } A_2 \text{ or } \ldots \text{ or } A_n \ldots)\), symbolically \( \bigcup_{i=1}^{\infty} A_i \)

or \( \bigvee_{i=1}^{\infty} A_i \).

3. not A, symbolically \( A^c \), \( C(A) \), \( \neg A \), or \( \sim A \)

4. (A and B) or C, symbolically \( (A \cap B) \cup C \) or \( (A \land B) \lor C \)

5. (A or B) and C, symbolically \( (A \cup B) \cap C \) or \( (A \lor B) \land C \)

6. If A then B, or A implies B; symbolically \( A \rightarrow B \) or \( A \implies B \)

For the classical logic (normal logic, Greek logic or the Aristotelian logic) whose algebra is known as the Boolean algebra (after the inventor of the algebra, Boole in the 19th century), the statements using “and” and “or,” 1 and 2 above, are assumed to be commutative, namely:

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1'. “A and B” is surely/certainly/definitely/deterministically well-defined and surely/certainly/definitely/deterministically is equal to “B and A”;

2'. “A or B” is surely/certainly/definitely/deterministically well-defined and equal to “B or A”;

whereas statements using a combination of “and” and “or”, 4 and 5 above, are distributive, namely:

4'. “(A and B) or C” is surely/certainly/definitely/deterministically equal to “(A or C) and (A or C)”;

5'. “(A or B) and C” is surely/certainly/definitely/deterministically equal to “(A and C) or (A and C)”.

Perhaps more importantly, especially with reference to the al-Ghazali’s logic, as far as the classical logic is concerned, the implicative statement or causal statement, statement 6 above, is understood as a “strong then,” or “strong implication” which means “surely/certainly/definitely/deterministically then or implies” which in turn means a strong consequence or a strong cause-and-effect principle. In other words, statement 6 means:

6'. “B is surely/certainly/definitely/deterministically caused by A” or “B is surely/certainly/definitely/deterministically a consequence of A”, or “A necessarily causes B”.

Lastly, implicit in the negative statement, negation of a statement, namely statement 3 above, it is assumed that:

3’. A statement and its negation are mutually exclusive, i.e., there is a law known as the law of excluded middle. In other words, a statement can only either true or false and, thus, the relevant system of logic is also known as the two-valued logic in order to differentiate from other n-valued logic (n=3,4,…), which was first developed by Tarski in the 1930’s, infinite-valued logic in probability theory (rigourously established in the 1930’s), and the possibility theory (first formulated by Zadeh in the 1960’s).

Now, in al-Ghazali’s reasoning or argumentation such strong commutative, distributive, and consequential (causative) laws are not allowed. Thus, the classical logic and its algebra (the Boolean algebra) briefly described above are inappropriate and not applicable to the al-Ghazali’s logic. But it seems that al-Ghazali and especially his followers until today still
accept the Aristotelian logic (and its algebra) without any sign of serious reservation except perhaps with a slight modification to a statement involved by adding the word “*insya Allah*” (God willing) instead of surely, certainly, definitely, or deterministically. In fact, more than a century ago, Homes already had an opinion that al-Ghazali himself had tried to reconcile his faith in Islam with the classical (the Greek/Aristotelian logic) through his other well-known writing, *al-Kimya al-Sa’adaht* (The Alchemy of Happiness). How proper is this attitude? Is it mathematically valid? The problem is of course the modeling of “*insya Allah*.” For a start, one would incline to interpret “*insya Allah*” as possible or probable (perhaps the most common translation to the original al-Ghazali’s word, *mumkin*), which means presumably to include that of “possible” in the possibility theory (based on the fuzzy statements by Zadeh in the 1960’s) and that “probable” in the classical probability theory (based on the Boolean algebra rigorously formulated by Kolmogorov in the 1930’s although the relevant probability concept was first introduced by Galileo in the 17th century) and for some even naively to include that of “probable” in the quantum probability (in atomic physics as formulated by Schroedinger in the 1920’s). However, it is well known that (since the early twentieth century) the Aristotelian logic is no longer valid the moment that the word “possible” is brought into the realm of the Aristotelian logic. In fact, there is a new logic known as the modal logic, which has been developed to replace it. Still another model for “*insya Allah*” is perhaps the expression “not necessary” because it is said that al-Ghazali rejects “the principle of necessary causal connection” in the Aristotelian logic and that al-Ghazali also used the terms *dharuraht* (improperly translated as ‘necessity’) and ‘aadaht (habit) in describing his theory of natural causality, whereas others may prefer the term “contingent” as a better translation for *mumkin*.

As far as the “probable” in the sense of a quantum mechanical statement is concerned, the situation is even worse since it is also well known that some quantum statements actually do not satisfy the Boolean algebra, in particular statements involving “and” and “or”. Hence, statement 4 and 5 above, do not satisfy the Boolean distributive laws (statement 4’ and 5’ above) at all. So for the moment let us exclude the quantum mechanical statements.

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17 See Mas, “Quiyas: A Study in Islamic Logic.”
18 See Rayan, “Al-Ghazâlî’s Use of the Terms Ḍarûrah and ‘Âdah in His Theory of Natural Causality.”
Let us examine the gist of the modal logic\(^{20}\) in order to see its inappropriateness to the algebraic structure of the al-Ghazali’s reasoning for the classical statements (the non-quantal mechanical statements), which was first developed axiomatically by Lewis in 1913 but only became an acceptable system of logic in the 1940’s, especially after the work of Kripke.

**The Incompatibility of the Modal Logic with al-Ghazali’s Reasoning**

Modal logic is the study of the deductive behavior of the expressions ‘it is necessary that’ and ‘it is possible that’ and ‘it is probable that.’ Aristotle put some thought on the logical nature of these statements and came to the conclusion that “possible is neither necessary nor impossible” and brought into his syllogism (known as modal syllogism) with some light into it but proved to be impractical even after Theophrastus (after Aristotle, i.e., the end of the third century BC) changed the meaning of “probable” into simply “not impossible” so that Aristotle’s modal syllogism becomes simpler. Muslim scholars during the Islamic Civilization, especially Ibn Sina, have substantially contributed to the improvement of the Aristotelian modal logic as it can be seen in one of his monumental works, *al-Isyarat wa al-Tanbihdaht*, which was translated by Inati.\(^{21}\) Obviously, the modal logic formulated by Ibn Sina was not fully understood by others during or even long after his time, partly due to its incompleteness and partly due to his contradictory conviction in his own theory in relation to his belief on the certainty of the laws of nature. It is not known when the first Western scholars (and who they were) realized that the Greek modal syllogism is of no use, but since Lukasiewicz (a logician, died in 1956), it has been a common knowledge that Aristotle’s modal syllogism is “incomprehensible due to its many faults and inconsistencies, and that there is no hope of finding a single consistent formal model for it.”\(^{22}\) Obviously, they do not know the work of Ibn Sina mentioned above, or they simply unjustly ignored it. This is another matter, which we do not intend to discuss here. Meanwhile, al-Ghazali must have had


rigorously challenged the Greek modal syllogism with his new theory of natural causality mentioned above. But however intense the concern of Muslim scholars were with regard to the modal statements, hence, Arabic grammar and logic, it looks as though the Muslims scholars were somehow just happy to use the Greek logic simply with some reservations. The world had to wait until early twentieth century before new axiomatic modal logic emerged from the Western logicians who were apparently not challenged by the al-Ghazali’s logic at all but simply reacted towards the state of the (Western) mathematical logic itself which has many unsolved paradoxes in particular “the problem of false premises imply many correct implications.” With the new modal logic, a partial solution to this problem has been obtained. However, in the present modal logic, the possibility and probability modalities are regarded as of the same status even though linguistically there is a subtle difference between the two terms and indeed different mathematical models have been established for them (“possibility” is for a fuzzy statement which is not an element of a Boolean algebra, whereas “probability” is for a crisp statement or classical/Aristotelian statement which is an element of a Boolean algebra).

Normally in modal logic, only two operators are introduced:

\[ \Box \text{ for Necessarily and } \Diamond \text{ for Possibly.} \]

Thus the expression “necessarily p” or “it is necessary p” is denoted by a prefixed “box” (\(\Box p\)); whereas a prefixed “diamond” (\(\Diamond p\)) denotes “possibly p” or “it is possible p.”

Further, it is assumed that:

“necessarily” is the same as the expression “not possible that not”
“possibly” is the same as the expression “not necessarily not”; or symbolically

\[ \Box p \text{ is equivalent to } \neg \Diamond \neg p \]
\[ \Diamond p \text{ is equivalent to } \neg \Box \neg p \]

This is known as the Axiom 0 in the modal logic. This is obtained from presumably a logical conclusion such as “It is possible that it will be an accident today if and only if it is not necessary that it will not be an accident today”. This axiom is also often stated in the following ways:

necessary is equivalent to not possibly false; and

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23 See Gyekye, “Al-Ghazali on Causation.”

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possible is equivalent to not necessarily false (regardless of whether it is actually true or actually false); or (reading the axiom from right to left and replacing “not p” by X)
“It is not possible that X” is logically equivalent to “It is necessary that not X”; and “It is not necessary that X” is logically equivalent to “It is possible that not X.”

Then it is found that several other axioms are needed to obtain a reasonably good system of modal logic, now known as the normal modal logic, such that one can prove that “If a statement is necessary that the statement is true”; “If a statement is necessary, then it is necessary that the statement is necessary”; and “If a statement is possible, then it is necessary that the statement is possible”. This modal logic gives the nature of “possible”, “necessary”, and “contingent” as follows:

contingent is equivalent to not necessarily false and not necessarily true (i.e., possible but not necessarily true). The words “possible” and “contingent” are considered as the two sorts of truth. The normal modal logic also gives the following important theorems:

T1.1. The necessity of A and the necessity of B are strongly equivalent to the necessity of A and B, or symbolically
\[ (\square A) \land (\square B) \leftrightarrow \square (A \land B). \]

T1.2. The possibility of A or the possibility of B is strongly equivalent to the possibility of A or B, or symbolically
\[ (\lozenge A) \lor (\lozenge B) \leftrightarrow \lozenge (A \lor B). \]

T2.1. If A strongly implies B, then the necessity of A strongly implies the necessity of B and the possibility of A strongly implies the possibility of B, or symbolically
\[ (A \rightarrow B) \rightarrow (\square A \rightarrow \square B) \text{ and } (\lozenge A \rightarrow \lozenge B). \]

T2.2. If the necessity of A strongly implies B, then the possibility of A strongly implies the possibility of B, or symbolically
\[ \square (A \rightarrow B) \rightarrow (\lozenge A \rightarrow \lozenge B). \]

T2.3. If the possibility of A strongly implies B, then the necessity of A strongly implies the necessity of B, or symbolically
\[ \lozenge (A \rightarrow B) \rightarrow (\square A \rightarrow \square B). \]
T3. It is not true that the necessity of A or B strongly implies the necessity of A or the necessity of B; and the possibility of A and the possibility of B strongly imply the possibility of A and B. Symbolically, these statements are as follows:

\[ \square(A \lor B) \rightarrow (\square A \lor \square B), \text{ and} \]
\[ (\Diamond A \land \Diamond B) \rightarrow \Diamond(A \land B), \text{ where } \rightarrow \text{ denotes “not strongly-implies.”} \]

Now, it is clear that, as far as the algebraic structure of a known modal logic is concerned, it is NOT just a Boolean algebra augmented with the “modal algebra” of the “box” and the “diamond.” For example, even though necessary statements are commutative with respect to (w.r.t) “and” (Theorem T1.1) but not w.r.t. “or” (Theorem T3); whereas possible statements are commutative w.r.t “or” (theorem T1.2) but not w.r.t “and” (theorem T3). Similarly, with the “not necessary statements,” they are only commutative w.r.t. “and” (by T1.1 and T3). Of course, the distributive laws in modal logic are invalid as well. Contingent statements are also noncommutative. The modal logic is a 3-valued logic because there is another state, the “contingent state,” other than possible and necessary.

The non-Boolean nature of the present modal logic is unsatisfactory especially since it is inconceivable to have the following with regard to the possible, not necessary, or contingent as a model of insya Allah:

\[ (\text{insya Allah } A) \text{ and/or (insya Allah } B) \neq (\text{insya Allah } B) \text{ and/or (insya Allah } A) \]

Many algebraists are happy to define a modal algebra as a Boolean algebra of ordinary naïve sets with an operator “necessary” which preserves “and” or also Boolean w.r.t “and” for “set of necessary sets of statements.” But it does not solve our problem of identifying a correct modal algebra for al-Ghazali’s logic.

More importantly, of course, we would like to have the “strict implication” or “strong implication” to be replaced by the “insya Allah implication” which the author was hoping to model by “possible” or “not necessary,” viz. “possible implication” or “not necessary implication.” However, none such statement is found in the present modal logic. The nearest statement in the modal logic in this regard is an implicative statement of the form given by “a possible statement strictly implies another possible statement” such as in the theorem T2 above. This is, of course, not sufficiently satisfactory.
Conclusion

Based on the discussion above, it is necessary to examine the exact meaning of mumkin (the Islamic possibility), dharuraht (the Islamic necessity), taqdyr (the Islamic fate), and insya Allah (the Islamic will of God) so that a new axiomatic modal logic could be formulated to improve or even replace the present algebraic structure which would be most suited for al-Ghazali’s logic. With regard to mumkin, there is an interesting Malay manuscript MS1659 (according to the code of the manuscript by Malaysian National Library, elaborated in the References under the subheading Manuscript) believed to be written by a well-known Malayonesian scholar in the 17th century ‘Abd al-Ra’uf Singkel, entitled Mutiara yang Putih (White Pearl), on page 137-146, in which the four types of mumkins namely mumkin mawjud, mumkin wajidwu anqadhy, mumkin sayyuwajid, and mumkin innahu lam yuwajid are introduced and explained. This manuscript, among other means, is perhaps relevant for obtaining some ideas towards solving this problem, namely, the problem of modelling the logical structure of insya Allah using mumkin.

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References

Azzam, S., ed., Islam and Contemporary Society (United Kingdom: Longman Group, 1982).


Malay Manuscript MS1659 (Kuala Lumpur: Perpustakaan Negara Malaysia, 17th Century).


